Mode-Based Tolerance Analysis in Multi-Station Assembly using Stream of Variation Model

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Outline

- Introduction
- Stream of Variation Analysis (SOVA) Methodology
  - SOVA Modeling for Multi-Station Assembly Processes
  - Mode based Tolerance Modeling
- Case Study
- Summary

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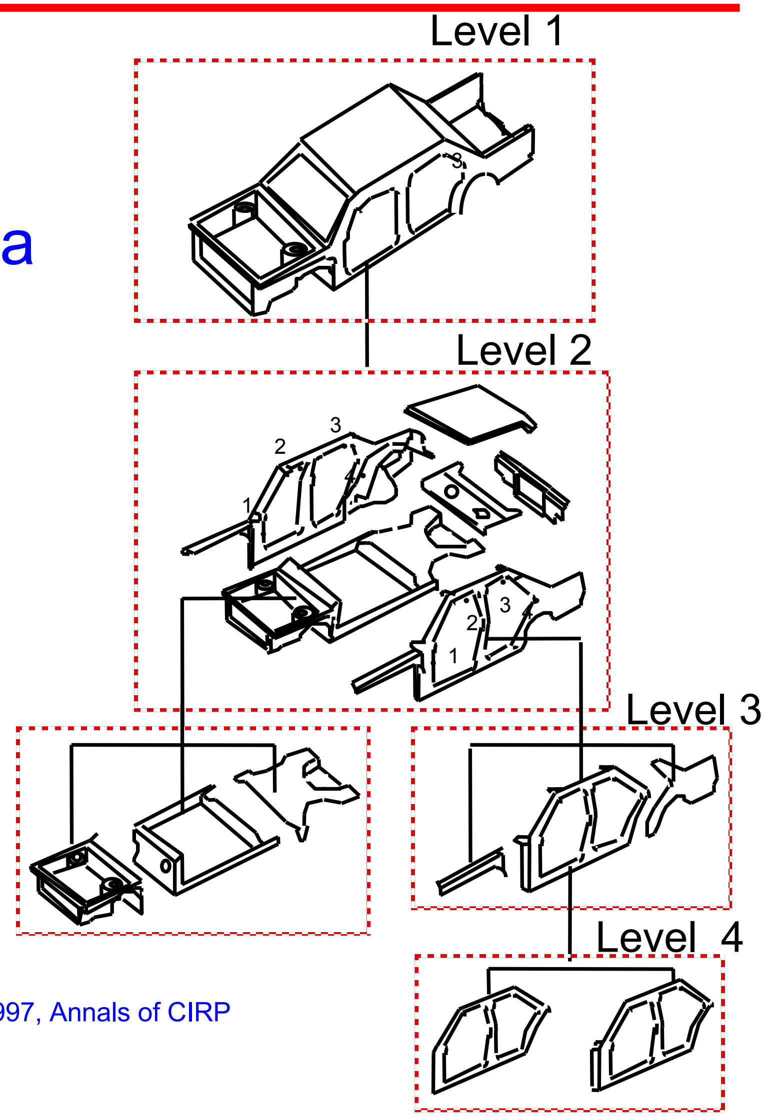
Multistage Manufacturing

- A complex manufacturing process that involves multiple stations or stages to produce a product.
- The output of one station/stage is the input of next station/stage.



Hu, 1997, Annals of CIRP

Example : Automotive Body Assembly

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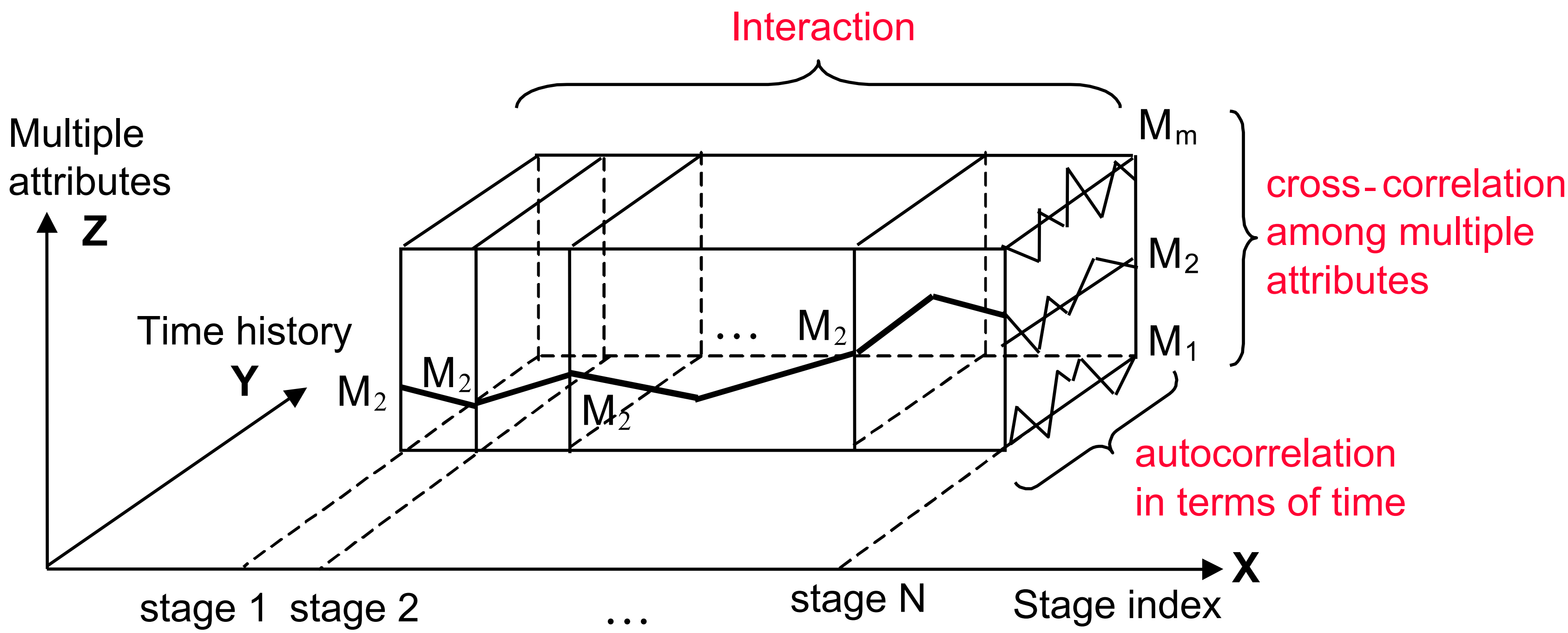
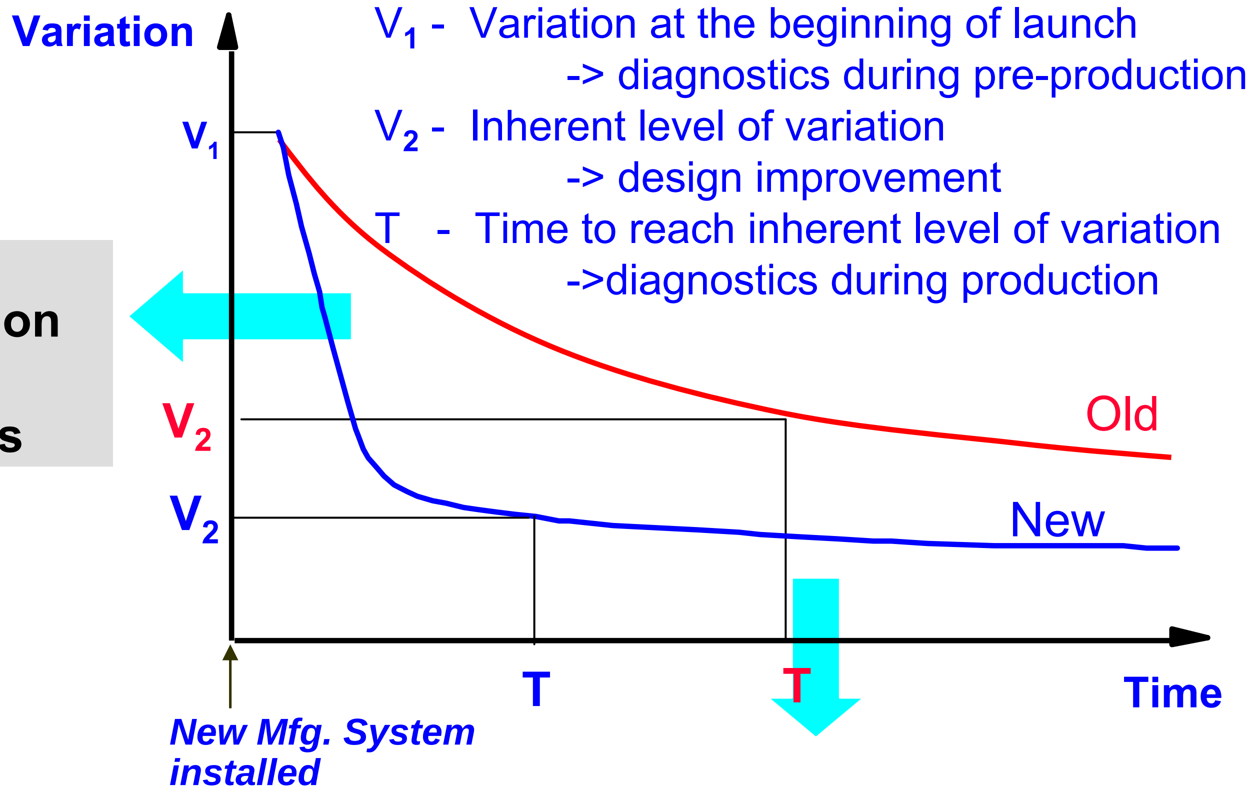
Stream of Variation

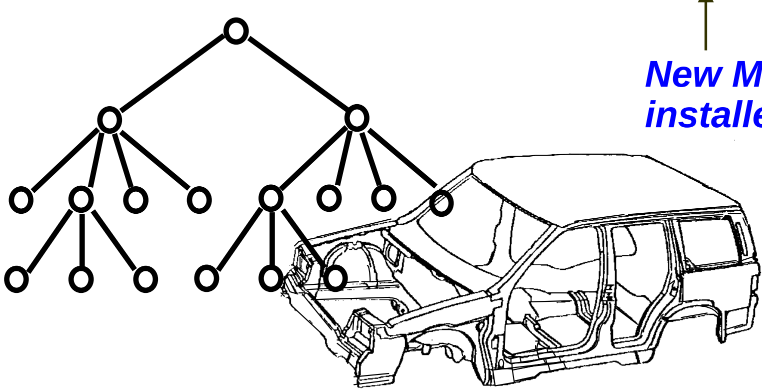


- **Interaction between manufacturing station/stages**
- **Accumulated process uncertainty**

Motivation



Design
Variation propagation
Tolerancing
System Robustness



Product & Process Models

Manufacturing
Variation propagation
Root cause identification

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Challenges in SOVA Modeling

- Related Works
  - Data Driven Model
    - Lawless et al. (1999)
    - Agrawal et al. (1999)
  - Engineering Model
    - Ceglarek et al. (1994, 1998)
    - Mantripragada and Whitney (1999)
    - Jin and Shi (1999)
    - Ding et al. (2000)
    - Camelio et al. (2002, 2004)
    - Cai et al. (2004)
- Gaps:
  - 2D instead of 3D
  - Incomplete tolerance Modeling

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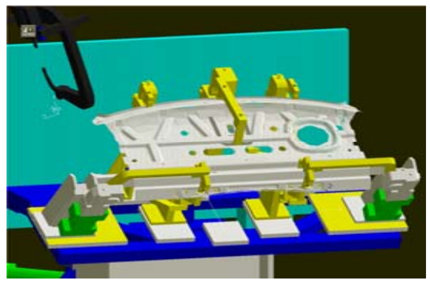
Elements in SOVA Modeling

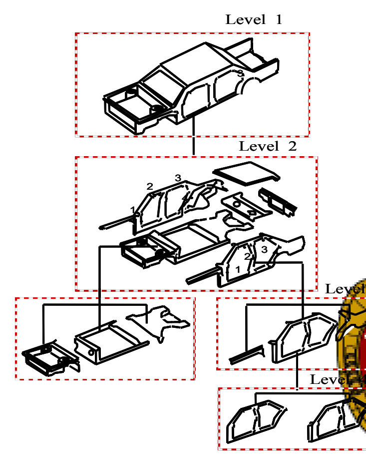
1. Part: Quality representation of features

2. Fixture Setup: Relationship between part and fixture

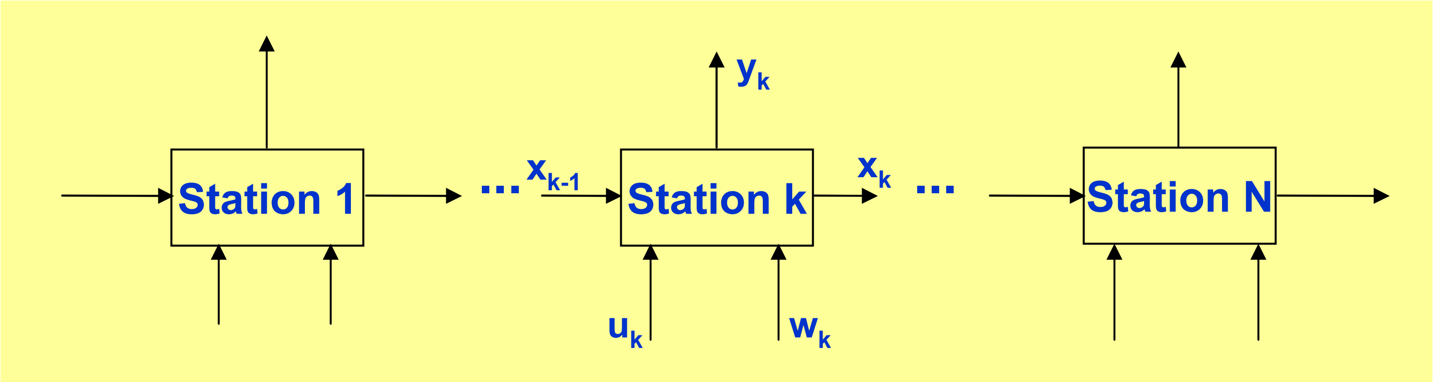


3. Assembly Operation: Relationship between fixture, part, and assembly tool

4. Assembly process/ sequence: Interaction between stations



SOVA Engine: State Space Modeling



- Variation Propagation Model
  - System Equation: $\mathbf{x_k = A_{k-1}\, x_{k-1} + B_k\, u_k + w_k}$ (k=1,2,…N)
  - Observation Equation: $\mathbf{y_k = C_k\, x_k + v_k}$
- State Space Modeling for Assembly Process:
  - The variation propagation can be modeled as a state-space system where operation stage plays the role of time

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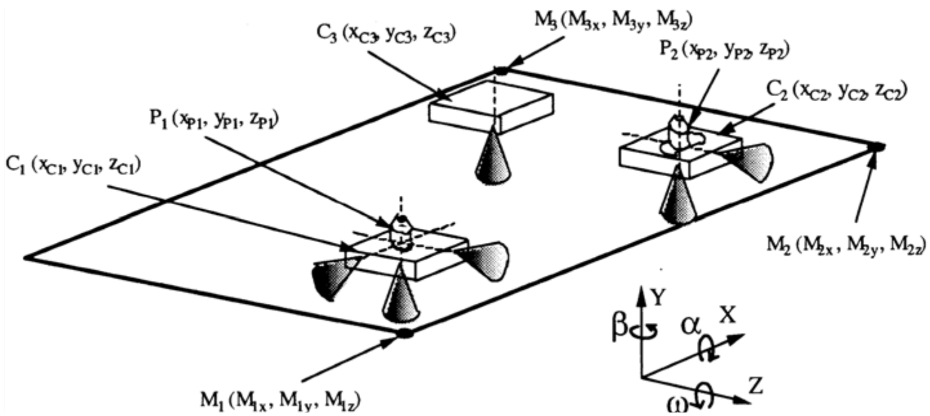
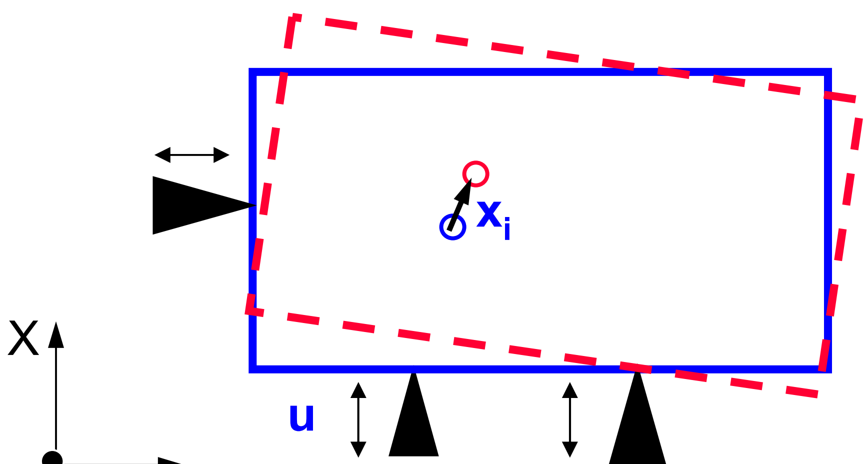
State Vector and Input

$$\mathbf{x}_k = \mathbf{A}_{k-1}\, \mathbf{x}_{k-1} + \mathbf{B}_k\, \mathbf{u}_k + \mathbf{w}_k \quad (k=1,2,\ldots N)$$



3-2-1 Fixture Locating Scheme

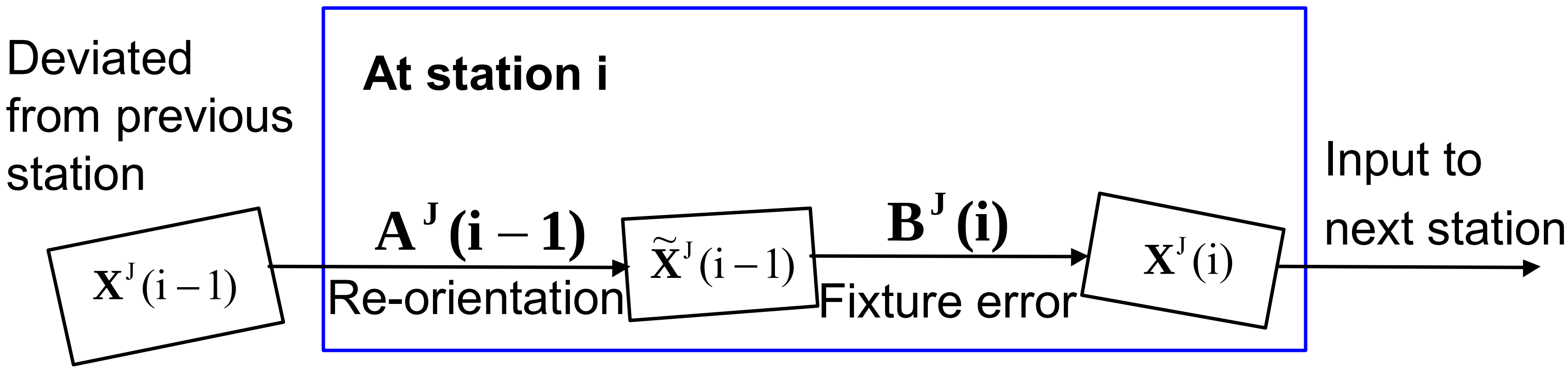


Part Deviation

- **u** : Fixture deviation vector
- $\mathbf{x}_i$: Part deviation vector
  - [δx, δy, δz, δα, δβ, δγ]’

Deviation Accumulation Between Stations

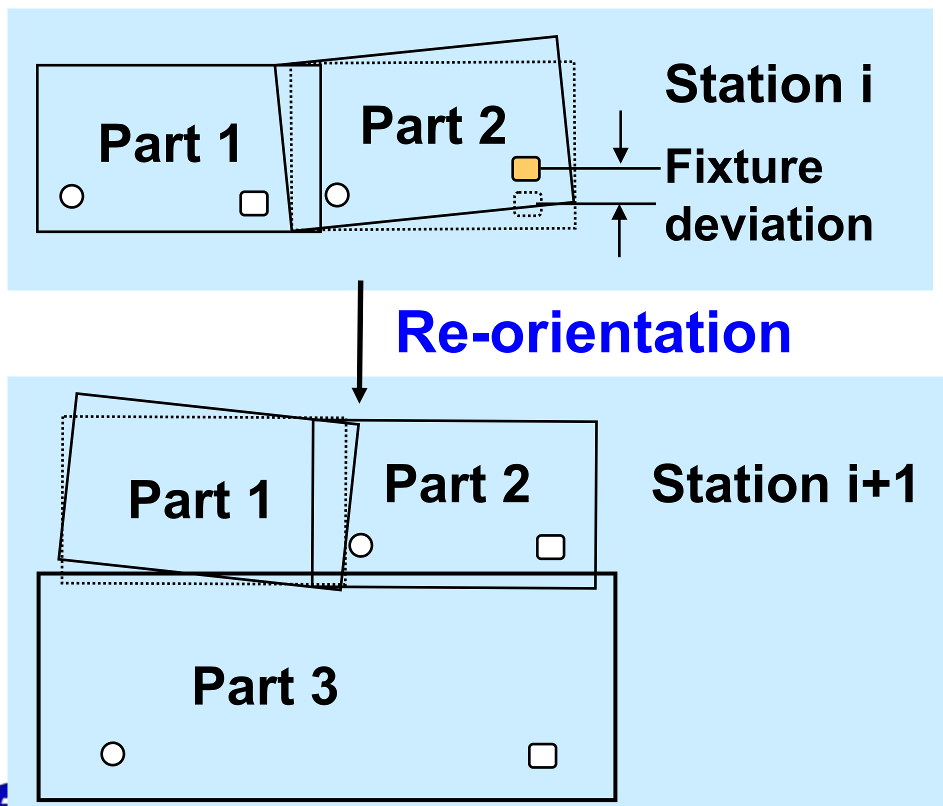
$$x_k = A_{k-1} x_{k-1} + B_k u_k + w_k \ (k=1,2,\ldots N)$$



Deviation accumulation on part J at station i

Station Transition

- $\mathbf{A_i}$: dynamic matrix, characterizes the assembly transition across stations.
- $\Phi_{k,i}=A_{k-1}A_{k-2}\ldots A_i$ is the state transition matrix.



- Adjacent stations

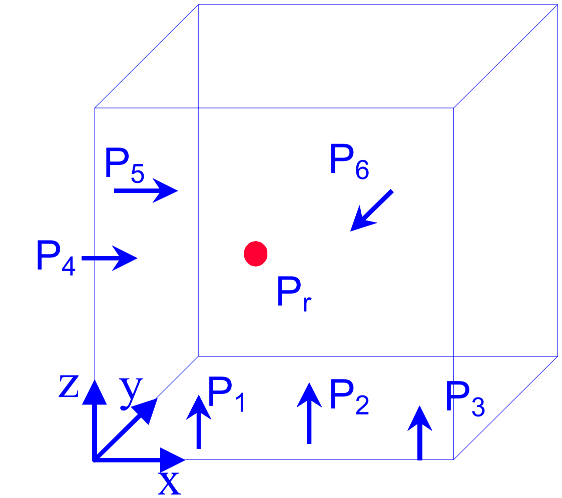
$$X_{i-1} \xrightarrow{A_{i-1}} X_i$$

- Multiple stations

$$X_i \xrightarrow{\Phi_{k,i}} X_k$$

SOVA Modeling: Rigid Part Assembly

- B_i: represents how the product quality is affected by the process error sources



Rigid Body 3-2-1 locating scheme

Part Deviation — **Fixture Layout Design** — **Error Sources**

$$\Delta P_r = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta\alpha \\ \Delta\beta \\ \Delta\gamma \end{bmatrix}_r = F_s \Delta f = \begin{bmatrix} \frac{D_{y5r}}{D_{y54}} & \frac{D_{yr4}}{D_{y54}} & 0 & \frac{D_{zr4}D_{y32}}{C} & \frac{D_{zr4}D_{y13}}{C} & \frac{D_{zr4}D_{y21}}{C} \\ \frac{D_{xr6}}{D_{y54}} & \frac{D_{x6r}}{D_{y54}} & 1 & \frac{D_{z6r}D_{x32}}{C} & \frac{D_{z6r}D_{x13}}{C} & \frac{D_{z6r}D_{x21}}{C} \\ 0 & 0 & 0 & E & F & G \\ 0 & 0 & 0 & \frac{D_{x32}}{C} & \frac{D_{x13}}{C} & \frac{D_{x21}}{C} \\ 0 & 0 & 0 & \frac{D_{y32}}{C} & \frac{D_{y13}}{C} & \frac{D_{y21}}{C} \\ \frac{1}{D_{y54}} & \frac{-1}{D_{y54}} & 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \Delta x_4 \\ \Delta x_5 \\ \Delta y_6 \\ \Delta z_1 \\ \Delta z_2 \\ \Delta z_3 \end{bmatrix},$$

where:

$$\begin{cases} D_{xij} = (X_i - X_j) \\ D_{yij} = (Y_i - Y_j) \\ C = D_{x21}D_{y31} - D_{x31}D_{y21} \\ E = 1 + (D_{x1r}D_{y32} + D_{y1r}D_{x23})/C \\ F = (-D_{x1r}D_{y31} + D_{y1r}D_{x31})/C \\ G = (D_{x1r}D_{y21} \quad D_{y1r}D_{x21})/C \end{cases}$$

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State Transition Model & Observation Equation

x(k) = A(k-1)x(k-1) + B(k)u(k) + w(k)   (k=1,2,…N.)

Re-orientation

$$A(i\text{-}1) = \begin{cases} 0_{6n_t \times 6n_t} \\ \begin{bmatrix} M(i) \\ 0_{6(n_t-k_{i-1})\times 6n_t} \end{bmatrix}_{6n_t\times 6n_t} + \begin{bmatrix} 0_{6k_{i-1}\times 6n_t} \\ R(i-1) \\ 0_{6(n_t-k_i)\times 6n_t} \end{bmatrix}_{6n_t\times 6n_t} \end{cases}$$

Within Single Station

$$B(i) = \begin{cases} \Lambda(i)_{6n_t\times m(i)} \\ \begin{bmatrix} M(P_{sr}(i), P_r)\overline{W}_0 \\ \Lambda(i) \end{bmatrix}_{6n_t\times m(i)} \end{cases}$$

w(k)

Unmodeled Error Term

Error Input:

$$U(i) = \begin{cases} \left\{ \overbrace{\Delta f'_1, \Delta\Omega'_{b1}}^{\text{Part 1}}, \overbrace{\Delta\Omega'_{a2}, \Delta f'_2, \Delta\Omega'_{b2}}^{\text{Part 2}}, \overbrace{\ldots, \quad \ldots}^{\cdots}, \overbrace{\Delta\Omega'_{ak_1}, \Delta f'_{k_1}}^{\text{Part } k_1} \right\} & \text{if} \quad i=1 \\ \left\{ \overbrace{\Delta f'_s, \Delta\Omega'_s}^{\text{Subassembly } s}, \overbrace{\Delta\Omega'_{a(k_{i-1}+1)}, \Delta f'_{k_{i-1}+1}, \Delta\Omega'_{b(k_{i-1}+1)}}^{\text{Part } k_{i-1}+1}, \overbrace{\ldots, \quad \ldots}^{\cdots}, \overbrace{\Delta\Omega'_{ak_i}, \Delta f'_{k_i}}^{\text{Part } k_i} \right\} & \text{if} \quad i>1 \end{cases}$$

y(k) = C(k) x(k) + v(k)   (k∈{1,2,…N})

y(k)

Observed KPC

Measurement Scheme

$$C(i) = \begin{bmatrix} \mathbf{C}^1(i) & \mathbf{0}^{6r^1\times 6} & \cdots & \mathbf{0}^{6r^1\times 6} \\ \mathbf{0}^{6r^2\times 6} & \mathbf{C}^2(i) & \cdots & \mathbf{0}^{6r^2\times 6} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}^{6r^{n_t}\times 6} & \mathbf{0}^{6r^{n_t}\times 6} & \cdots & \mathbf{C}^{n_t}(i) \end{bmatrix}_{(6\sum_{J=1}^{n_t} r^J)\times 6n_t}$$

v(k)

Observation Noise

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Variation Propagation Model

- Input-Output Model

$$\mathbf{y} = \sum_{i=0}^{N} \gamma(i)\mathbf{u}(i) + \boldsymbol{\varepsilon}$$

where

$$\begin{cases} \gamma(i) = C\Phi(N,i)B(i) \\ \gamma(0) = C\Phi(N,0) \\ \Phi(N,i) = A(N-1)A(N-2)\ldots A(i) \\ \boldsymbol{\varepsilon} = \sum_{i=1}^{N} C\Phi(N,i)v(i) + w \end{cases}$$

- Fault - Quality Model for Statistical Analysis:

$$\underbrace{\mathbf{y}_{m\times 1}}_{\text{Measurement (KPC)}} = \underbrace{\Gamma_{m\times n}}_{\text{SOVA matrix}} \cdot \underbrace{\mathbf{u}_{n\times 1}}_{\text{Error Sources (KCC)}} + \boldsymbol{\varepsilon}$$

m: number of measurements
n: number of error sources

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Tolerance Modeling: Tolerance Modes

$$\mathbf{y} = \Gamma \cdot \mathbf{u}$$

$$\Downarrow$$

$$\text{cov}(y) = \Gamma \bullet \text{cov}(u) \bullet \Gamma' = \Gamma \bullet \begin{vmatrix} \text{var}(u_1) & \text{cov}(u_1,u_2) \\ \text{cov}(u_1,u_2) & \text{var}(u_2) \end{vmatrix} \bullet \Gamma'$$

Independent Tolerance Mode: $\text{cov}(u_1,u_2) = 0$

Group Tolerance Mode: $\text{cov}(u_1,u_2) = u_1 u_2/36$

Composite Tolerance Mode: $\text{cov}(u_1,u_2) =$???

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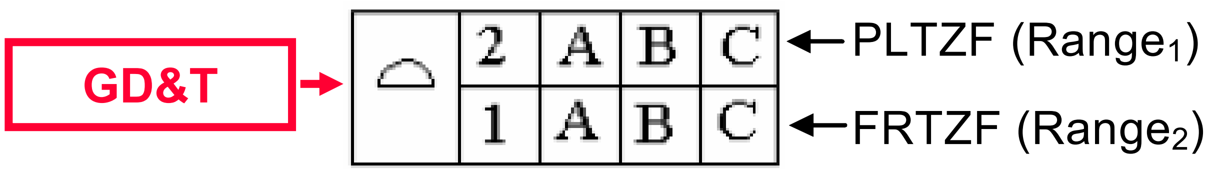
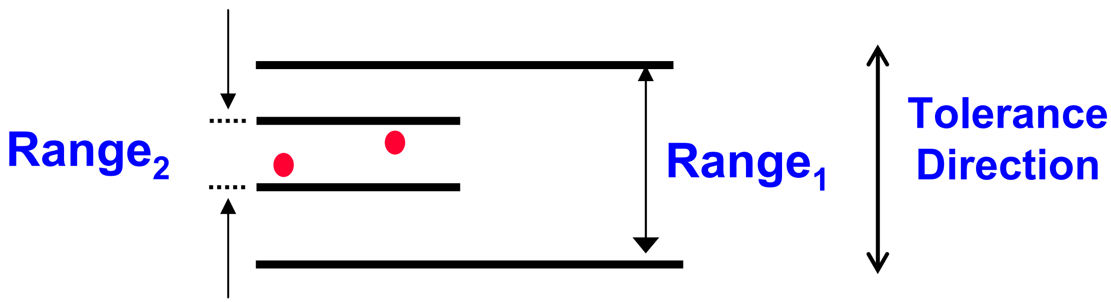
Tolerance Modeling: Composite Tolerance





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Tolerance Modeling: Composite Tolerance

Factor Analysis → Let u' represent the latent error sources, which are uncorrelated,

$\mathbf{u' = | u_1' \quad u_2' |'}$ $\mathbf{cov(u_1', u_2') = 0}$ $\mathbf{u = Qu'}$ Q is a rotational matrix

- Q is orthonormal
- Definition of composite tolerance → $\mathbf{var(u_1 - u_2) = 2 \bullet (range_2/6)^2}$

Covariance matrix →

$$\begin{vmatrix} \mathrm{var}(u_1) & \mathrm{cov}(u_1, u_2) \\ \mathrm{cov}(u_1, u_2) & \mathrm{var}(u_2) \end{vmatrix} = (\mathrm{range}_1/6)^2 \bullet \begin{vmatrix} 1 & 1-k \\ 1-k & 1 \end{vmatrix}$$

with $\mathbf{k = (range_2/range_1)^2}$

Case Study

120 Parts
42 Stations
111 Assembly Operations
1169 Tolerances
390 Measurements
Γ: 390×1169

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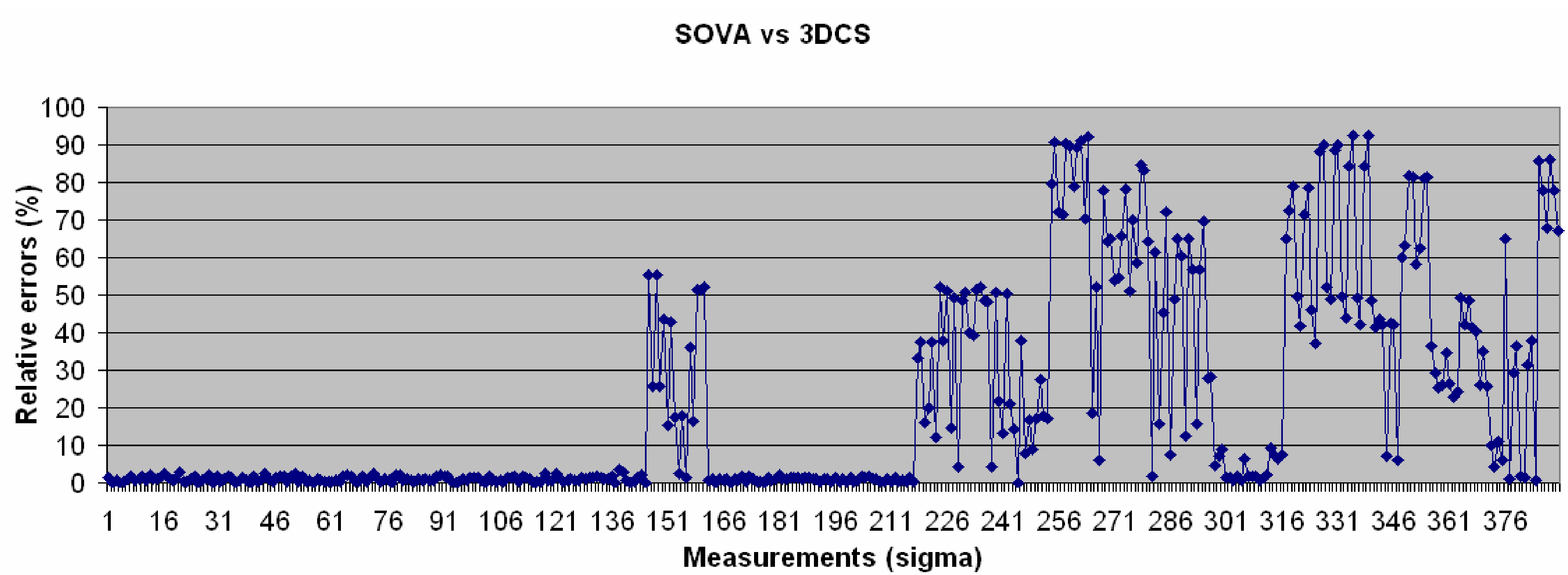
Case Study

WITHOUT consideration of tolerance modes

Max: 92.39%
Min: 0.01%
Avg: 20.65%



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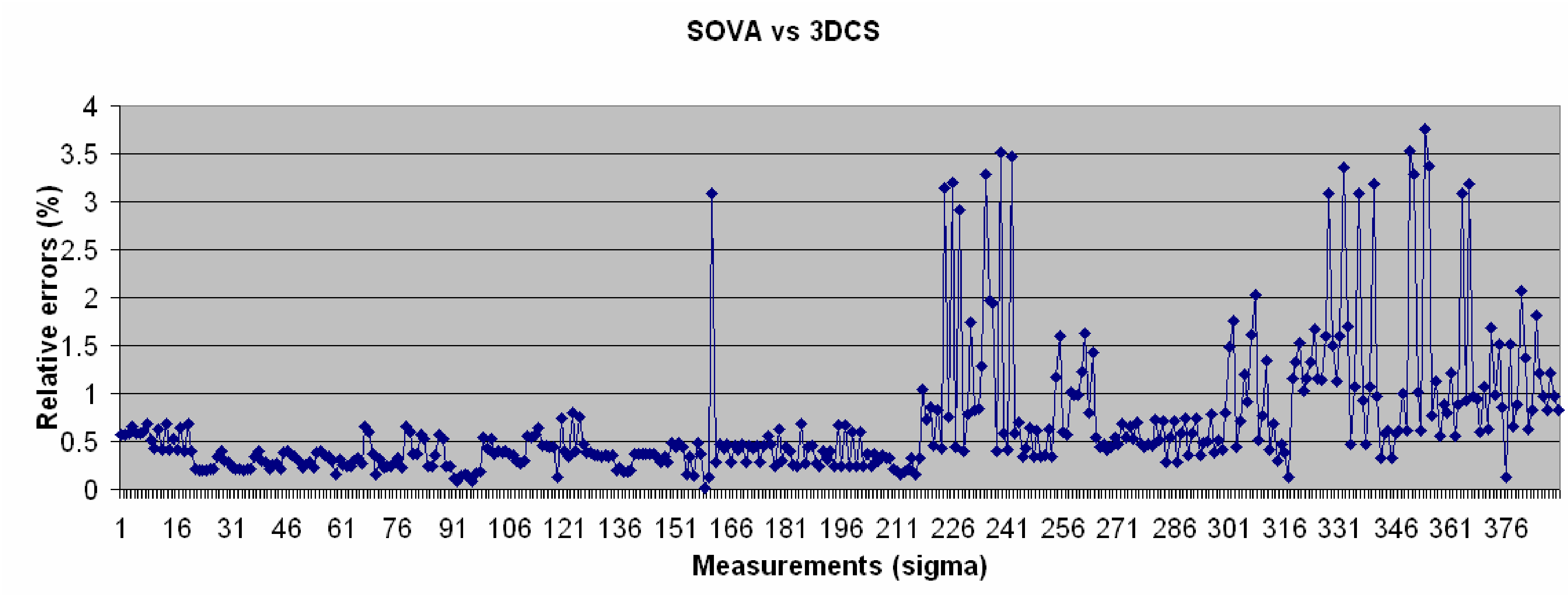
Case Study

WITH consideration of tolerance modes

Max: 3.75%
Min: 0.01%
Avg: 0.68%



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Summary

- Enhanced SOVA modeling
- Complete mode-based tolerance modeling
- Successfully applied in a real application
- The developed model can be applied for quality improvement in both design and manufacturing stages

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Thank You!

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